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QNET Rotary Inverted Pendulum Trainer for NI ELVIS

Developed by Quanser

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1 INTRODUCTION

Regulation and servo problems are very common, but feedback can be used in many other useful ways. The name task-based control is used as a common classification of a wide variety of problems. For instance, stabilization of an unstable system can be considered a task-based problem. However, it is a borderline example since it can also be viewed as a regulation problem. The Segway transporter is a typical example where stabilization is a key task. In that case stabilization is also merged with the steering functions. Other examples are damping of a swinging load on a crane, stabilization of a rocket during take-off, and the human posturing systems. There are many examples of task-based control in aerospace such as automatic landing and orbit transfer of satellites. Robotics is a rich field for task-based control with challenges such as collision avoidance, motion planning, and vision based control. Task-based control is typically more complicated than regulation and servoing but they may contain servo and regulation functions as sub-tasks. We have chosen the rotary pendulum system to illustrate task-based control

The QNET rotary inverted pendulum trainer is shown in Figure 1.1. The motor is mounted vertically in a metal chamber. An L-shaped arm is connected to the motor shaft and pivots between ± 180 degrees. A pendulum is suspended on a horizontal axis at the end of the arm. The pendulum angle is measured by an encoder. The control variable is the input voltage to the pulse-width modulated amplifier that drives the motor. The output variables are the angle of the pendulum and the angle of the motor.

Figure 1.1: QNET rotary inverted pendulum trainer (ROTPENT)

There are three experiments: simple modeling, inverted pendulum balance control, and swing-up control. The experiments can be performed independently.

Topics Covered

- Modeling the pendulum
- Balance control (via state-feedback)

- Control optimization (LQR)
- Friction compensation
- Energy control
- Hybrid control

Prerequisites
In order to successfully carry out this laboratory, the user should be familiar with the following:

- Transfer function fundamentals, e.g. obtaining a transfer function from a differential equation.
- Using LabVIEW® to run VIs.

2 SIMPLE MODELING

2.1 Background

This experiment illustrates some control tasks for gantry cranes. The gantry is a moving platform or trolley that transports the crane about the factory floor or harbor. The load hangs from the crane using wires and is moved by the gantry crane. Typically the problem is to move the load quickly and move it to the correct position. The fast motion necessary for production makes it more difficult to move the load to the correct location given the swinging motions of the crane. This problem can be mimicked using the rotary pendulum system by viewing the tip of the L-shaped arm as the moving trolley and the pendulum tip as the load being carried.

In this experiment we will begin by modeling the system and determine strategies to dampen the oscillations of the system.

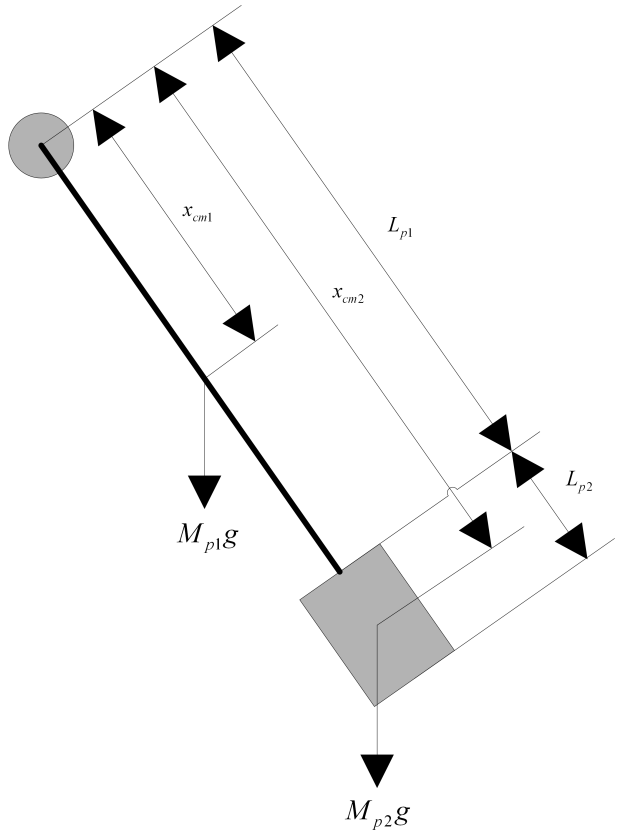

Figure 2.1: Free-body diagrams of pendulum assembly

Figure 2.1 shows the free-body diagram of the pendulum assembly that is composed of two rigid bodies: the pendulum link with mass M_{p1} and length L_{p1}, and the pendulum weight with mass M_{p2} and a length L_{p2}. The center of mass of the the pendulum link and the pendulum weight are calculated separately using the general expression

$$x_{cm} = \frac{\int p\, x\, dx}{\int p\, dx}$$

where x is the linear distance from the pivot axis and p is the density of the body. The circle in the top-left corner of Figure 2.1 represents the axis of rotation or the pivot axis that goes into the page.

The pendulum system is then expressed as one rigid body with a single center of mass, as shown in Figure 2.2.

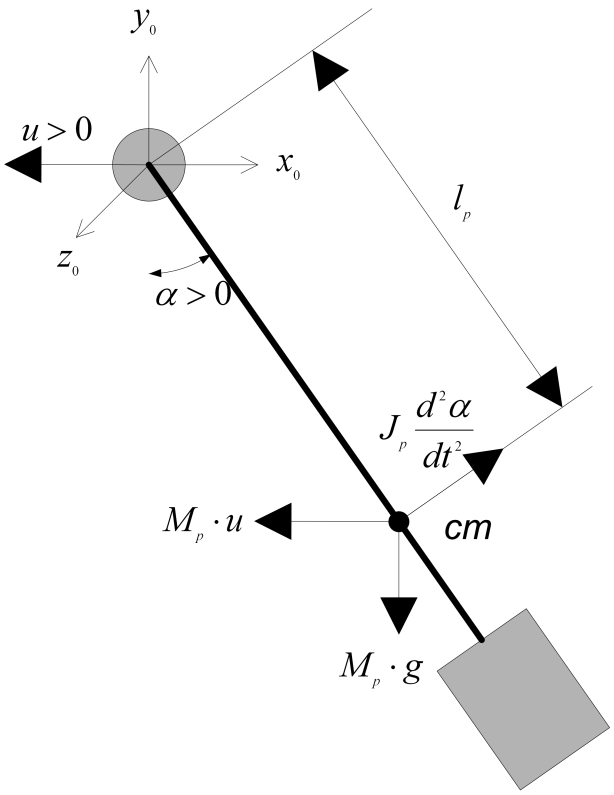

Figure 2.2: Free-body diagram of composite pendulum

The center of mass of a composite object that contains n bodies can be calculated using

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_{cm,i}}{\sum_{i=1}^{n} m_i}$$

where $x_{cm,i}$ is the known center of mass of body i and m_i is the mass of body i.

From the free-body diagram in Figure 2.2, the resulting nonlinear equation of motion of the pendulum is

$$J_p \ddot{\alpha}(t) = M_p \, g \, l_p \sin \alpha(t) + M_p \, u \, l_p \cos \alpha(t) \tag{2.1}$$

where J_p is the moment of inertia of the pendulum at the pivot axis z_0, M_p is the total mass of the pendulum assembly, u is the linear acceleration of the pivot axis, and l_p is the center of mass position as depicted in Figure 2.2. Thus as the pivot accelerates towards the left the inertia of the pendulum causes it to swing upwards while the gravitation force $M_p g$ and the applied force $M_p u$ (the left-hand terms in Equation 2.1) pull the pendulum downwards.

The moment of inertia of the pendulum can be found experimentally. Assuming the pendulum is unactuated, linearizing Equation 2.1 and solving for the differential equation gives the expression

$$J_p = \frac{M_p \, g \, l_p}{4 f^2 \pi^2} \tag{2.2}$$

where f is the measured frequency of the pendulum as the arm remains rigid. The frequency is calculated using

$$f = \frac{n_{cyc}}{\Delta t} \tag{2.3}$$

where n_{cyc} is the number of cycles and Δt is the duration of these cycles. Alternatively, J_p can be calculated using the moment of inertia expression

$$J = \int r^2 dm \tag{2.4}$$

where r is the perpendicular distance between the element mass, dm, and the axis of rotation.

In addition to finding the moment of inertia, this laboratory investigates the stiction that is present in the system. The rotor of the DC motor that moves the ROTPEN system requires a certain amount of current to begin moving. In addition, the mass from the pendulum system requires even more current to actually begin moving the system. The friction is particularly severe for velocities around zero because friction changes sign with the direction of rotation.

See Wikipedia for more information on: center of mass, inertia, pendulum, and friction.

2.2 Simple Modeling Virtual Instrument

The virtual instrument for studying the physics of the pendulum when in the gantry configuration is shown in Figure 2.3.

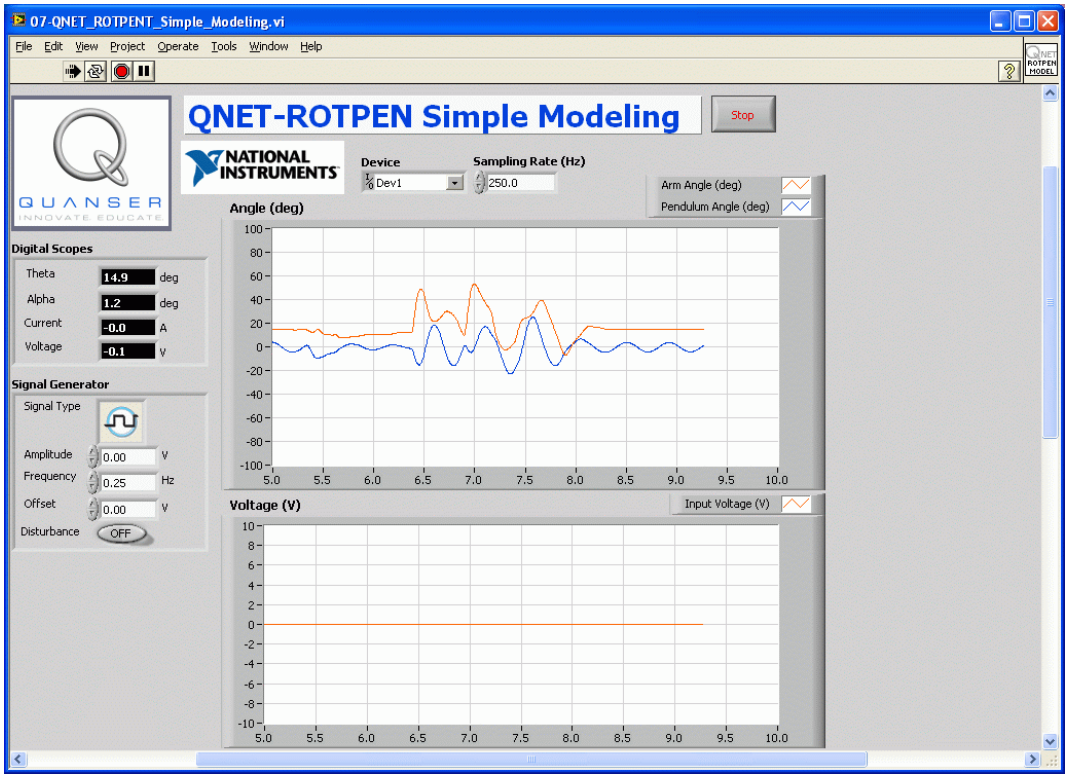

Figure 2.3: LabVIEW VI for modeling QNET rotary pendulum.

2.3 Damping [15 min]

1. Ensure the QNET ROTPENT Simple Modeling VI is open and configured as described in Section 2.2. **Make sure the correct *Device* is chosen.**
2. Run the QNET_ROTPENT_Simple_Modeling.vi shown in Figure 2.3.
3. Hold the arm of the rotary pendulum system stationary and manually perturb the pendulum.
4. While still holding the arm, examine the response of *Pendulum Angle (deg)* in the *Angle (deg)* scope. This is the response from the **pendulum** system.
5. Repeat 3 above but release the arm after several swings.

6. Examine the *Pendulum Angle (deg)* response when the arm is not fixed. This is the response from the **rotary pendulum** system. Given the response from the **pendulum** and **rotary pendulum** system, which converges faster towards angle zero? Why does one system dampen faster than the other?
7. Stop the VI by clicking on the *Stop* button.

2.4 Friction [15 min]

1. Run the QNET_ROTPENT_Simple_Modeling.vi.
2. In the *Signal Generator* section set
   - Amplitude = 0 V
   - Frequency = 0.25 Hz
   - Offset = 0.0 V
3. Change the *Offset* in steps of 0.10 V until the pendulum begins moving. Record the voltage at which the pendulum moved.
4. Repeat Step 3 above for steps of -0.10 V.
5. Enter the positive, V_{fp}, and negative voltage, V_{fn}, values needed to get the pendulum moving. Why does the motor need a certain amount of voltage to get the motor shaft moving?
6. Stop the VI by clicking on the *Stop* button.

2.5 Moment of Inertia [30 min]

2.5.1 Pre-Lab Questions

1. Find the moment of inertia acting about the pendulum pivot using the free-body diagram. Make sure you evaluate numerically using the parameters defined in the QNET ROTPEN User Manual ([2]).

2.5.2 In-Lab Exercises

1. Run the QNET_ROTPENT_Simple_Modeling.vi.
2. In the *Signal Generator* section set
   - Amplitude = 1.0 V
   - Frequency = 0.25 Hz
   - Offset = 0.0 V
3. Click on the *Disturbance* toggle switch to perturb the pendulum and measure the amount of time it takes for the pendulum to swing back-and-forth in a few cycles (e.g., 4 cycles).
4. Find the frequency and moment of inertia of the pendulum using the observed results. See Section 2.1 to see how to calculate the inertia experimentally.
5. Compare the moment of inertia calculated analytically in Exercise 1 and the moment of inertia found experimentally. Is there a large discrepancy between them?
6. Stop the VI by clicking on the *Stop* button.

2.6 Results

Fill out Table 1 with your answers from above.

Description	Symbol	Value	Unit
Section 2.4: Friction			
Positive Coulomb Friction Voltage	V_{fp}		V
Negative Coulomb Friction Voltage	V_{fn}		V
Section 2.5: Moment of Inertia			
Calculated inertia	J_p		kg · m^2
Experimentally found inertia	$J_{p,exp}$		kg · m^2

Table 1: QNET ROTPENT Modeling results summary

3 BALANCE CONTROL DESIGN

3.1 Background

A rich collection of methods for finding parameters of control strategies have been developed. Several of them have also been packaged in tools that are relatively easy to use. Linear Quadratic Regulator (LQR) theory is a technique that is suitable for finding the parameters of the balancing controller in Equation 4.1 in Section 4. Given that the equations of motion of the system can be described in the form

$$\dot{x} = Ax + Bu$$

the LQR algorithm computes a control task, u, to minimize the criterion

$$J = \int_0^\infty x(t)^T Q x(t) + u(t)^T R u(t) dt$$

The matrix Q defines the penalty on the state variable and the matrix R defines the penalty on the control actions. Thus when Q is made larger, the controller must work harder to minimize the cost function and the resulting control gain will be larger. In our case the state vector x is defined

$$x = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix}^T$$

Since there is only one control variable, R is a scalar and the control strategy used to minimize cost function J is given by

$$u = -K(x - x_r) = -k_{p,\theta}(\theta - \theta_r) - k_{p,\alpha}(\alpha - \pi) - k_{d,\theta}\dot{\theta} - k_{d,\alpha}\dot{\alpha}.$$

The LQR theory has been packaged in the LabVIEW® *Control Design and Simulation Module*. Thus given a model of the system in the form of the state-space matrices A and B and the weighting matrices Q and R, the LQR function in the *Control Design* Toolkit computes the feedback control gain automatically. In this experiment, the model is already available. In the laboratory, the effect of changing the Q weighting matrix while R is fixed to 1 on the cost function J will be explored.

See Wikipedia for more information on optimal control.

3.2 Balance Control Design VI

The QNET_ROTPENT_Control_Design VI has three tabs. Each tab is explained in the following sections.

3.2.1 Symbolic Model Tab

The *Symbolic Model* tab shown in Figure 3.1 is used to setup the QNET rotary pendulum model.

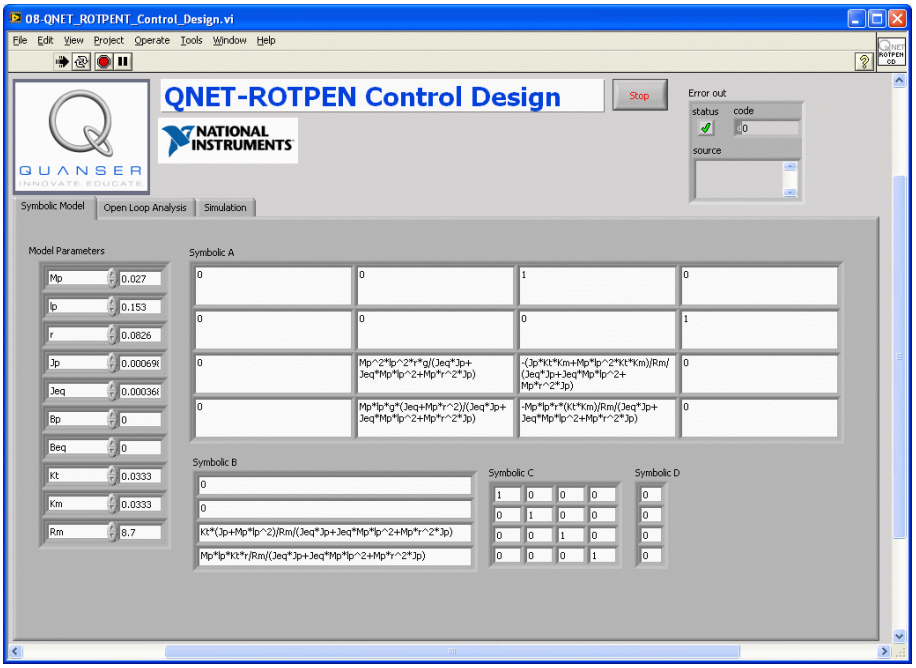

Figure 3.1: LabVIEW VI to generate state-space model of QNET rotary pendulum.

3.2.2 Open Loop Analysis Tab

The *Open Loop Analysis* tab on the VI is used to analyze the open loop stability of the QNET rotary pendulum system, shown in Figure 3.2.

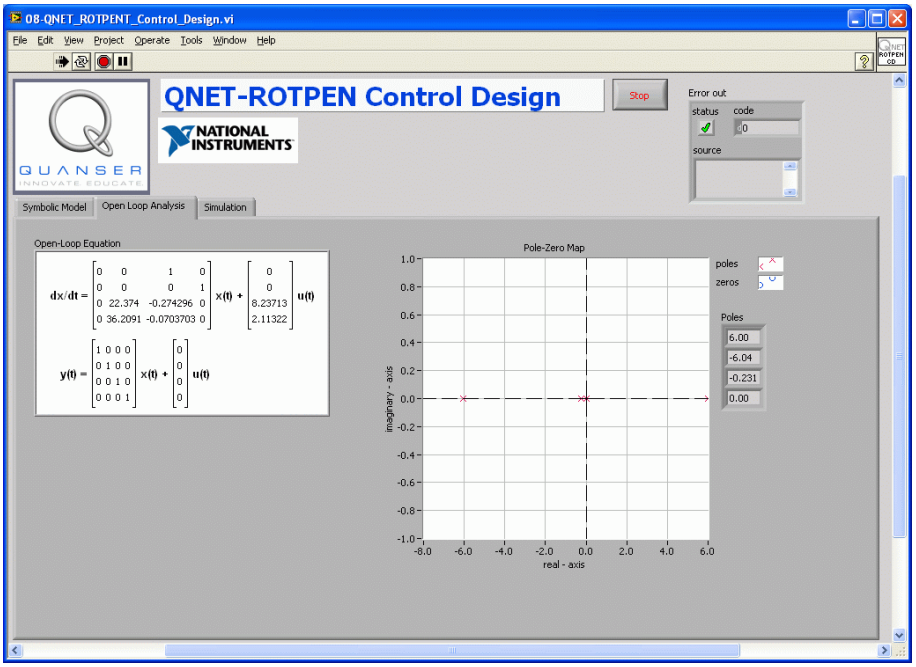

Figure 3.2: LabVIEW VI used to analyze open loop stability of QNET rotary pendulum system.

3.2.3 Simulation Tab

On the *Simulation* tab shown in Figure 3.3, users can generate the balance control gains for the QNET rotary pendulum system using LQR and simulate the closed-loop system.

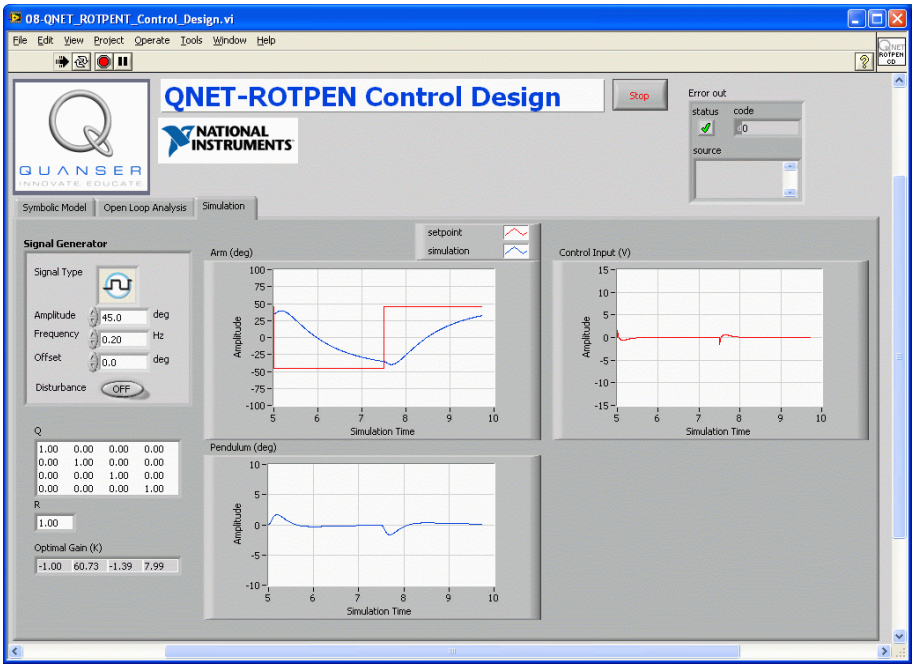

Figure 3.3: LabVIEW VI for QNET rotary pendulum balance control design.

3.3 Model Analysis [20 min]

1. Open the QNET_ROTPENT_Control_Design.vi.
2. Run the QNET_ROTPENT_Control_Design.vi. The front panel of the VI shown in Figure 3.1.
3. Select the *Symbolic Model* tab.
4. The *Model Parameters* array includes all the rotary pendulum modeling variables that are used in the state-space matrices A, B, C, and D.
5. Select the *Open Loop Analysis* tab, shown in Figure 3.2.
6. This shows the numerical linear state-space model and a pole-zero plot of the open-loop inverted pendulum system. What do you notice about the location of the open-loop poles? How does that affect the system? **Recommended**: In the *Model Parameters* section, it is recommended to enter the pendulum moment of inertia, *Jp*, be determined experimentally in Section 2.5.
7. In the *Symbolic Model* tab, set the pendulum moment of inertia, *Jp*, to 1.0×10^{-5} kg $\cdot$ m^2.
8. Select the *Open Loop Analysis* tab. How did the locations of the open-loop poles change with the new inertia? Enter the pole locations of each system with a different moment of inertia. Are the changes of having a pendulum with a lower inertia as expected?
9. Reset the pendulum moment of inertia, *Jp*, back to 1.77×10^{-4} kg $\cdot$ m^2.
10. Stop the VI by clicking on the *Stop* button.

3.4 Control Design and Simulation [45 min]

1. Open the QNET_ROTPENT_Control_Design.vi.
2. Select the *Simulation* tab.
3. Run the VI. The VI running is shown Figure 3.3.
4. In the *Signal Generator* section set:
   - Amplitude = 45.0 deg
   - Frequency = 0.20 Hz
   - Offset = 0.0 deg
5. Set the *Q* and *R* LQR weighting matrices to the following:
   - Q(1,1) = 10, i.e., set first element of Q matrix to 10
   - R = 1

   Changing the *Q* matrix generates a new control gain.
6. The arm reference (in red) and simulated arm response (in blue) are shown in the *Arm (deg)* scope. How did the arm response change? How did the pendulum response change in the *Pendulum (deg)* scope.
7. Set the third element in the *Q* matrix to 0, i.e., Q(3,3) = 0.
8. Examine and describe the change in the *Arm (deg)* and *Pendulum (deg)* scope.
9. By varying the diagonal elements of the *Q* matrix, design a balance controller that adheres to the following specifications:
   - Arm peak time less than 0.75 s: $t_p \leq 0.75$ s.
   - Motor voltage peak less than $\pm$ 12.5 V: $|V_m| \leq 12.5$ V.
   - Pendulum angle less than 10.0 deg: $|\alpha| \leq 10.0$ deg.

   Record the *Q* and *R* matrices along with the control gain used to meet the specifications in your report.
10. Attach the responses from the *Arm (deg)*, *Pendulum (deg)*, and *Control Input (V)* scopes when using your designed balance controller. Does it satisfy the specifications?
11. Stop the VI by clicking on the *Stop* button.

4 BALANCE CONTROL IMPLEMENTATION

4.1 Background

Balancing is a common control task. In this experiment we will find control strategies that balance the pendulum in the upright position while maintaining a desired position of the arm. When balancing the system the pendulum angle, α, is small and balancing can be accomplished simply with a PD controller. If we are also interested in keeping the arm in a fixed position a feedback from the arm position will also be introduced. The control law can then be expressed as

$$u = -k_{p,\theta}(\theta - \theta_r) - k_{p,\alpha}(\alpha - \pi) - k_{d,\theta}\dot{\theta} - k_{d,\alpha}\dot{\alpha} \tag{4.1}$$

where $k_{p,\theta}$ is the arm angle proportional gain, $k_{p,\alpha}$ is the pendulum angle proportional gain, $k_{d,\theta}$ is the arm angle derivative gain, and $k_{d,\alpha}$ is the pendulum angle derivative gain. The desired angle of the arm is denoted by θ_r and there is no reference for the pendulum angle because the desired position is zero.

There are many different ways to find the controller parameters. As discussed in Section 3.1, one method is based on LQR-optimal control. Initially, however, the behaviour of the system will be explored using default parameters.

When balancing the pendulum over a fixed point, the arm tends to oscillate about that reference because of the friction present in the motor. Due to friction, the motor will not move until the control signal is sufficiently large and the generated torque is larger than the stiction (see Section 2.1 for more details). This means that the pendulum has to fall a certain angle before the motor moves and the net result is an oscillating motion.

Friction can be compensated by introducing a Dither signal at the input voltage of the DC motor. The Dither signal used has the form

$$V_d = A_d \sin f_d t + V_{d0}$$

where A_d is the voltage amplitude, f_d is the sinusoid frequency, and V_{d0} is the offset voltage of the signal.

See Wikipedia for more information on PID and friction.

4.2 Balance Control VI

The virtual instrument used to run the balance controller (and the swing-up, shown later) on the QNET rotary pendulum system is shown in Figure 4.1.

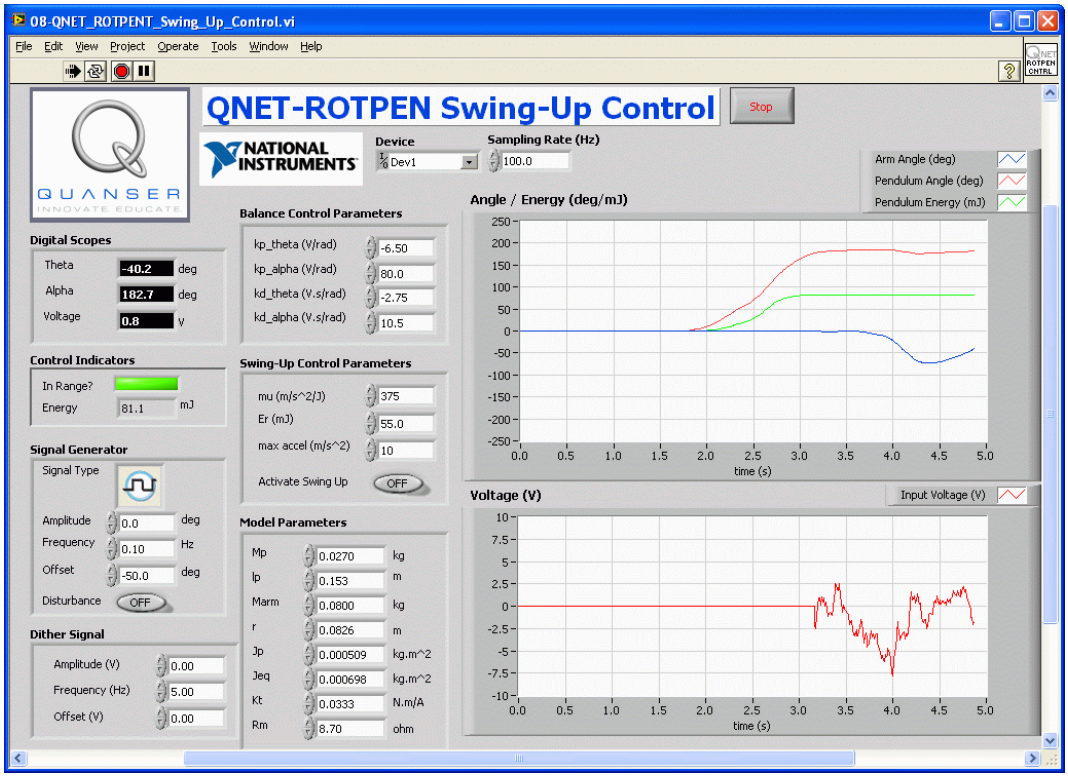

Figure 4.1: LabVIEW VI for QNET rotary pendulum balancing control (and swing-up).

4.3 Default Balance Control [30 min]

1. Open the QNET_ROTPENT_Swing_Up_Control.vi and ensure it is configured as described in Section 6. **Make sure the correct *Device* is chosen.**
2. Run the QNET_ROTPENT_Swing_Up_Control.vi. The VI should appear similarity as shown in Figure 4.1.
3. In the *Signal Generator* section set:
   - Amplitude = 0.0 deg
   - Frequency = 0.10 Hz
   - Offset = 0.0 deg
4. In the *Balance Control Parameters* section set:
   - kp_theta = -6.5 V/rad
   - kp_alpha = 80 V/rad
   - kd_theta = -2.75 V/(rad/s)
   - kd_alpha = 10.5 V/(rad/s)
5. In the *Swing-Up Control Parameters* section set:
   - mu = 55 m/s2/J
   - Er = 20.0 mJ
   - max accel = 10 m/s2
   - Activate Swing-Up = OFF (de-pressed)
6. Adjust the *Angle/Energy (deg/mJ)* scope scales to see between -250 and 250 (see Reference [2] for help).

7. Manually rotate the pendulum in the upright position until the *In Range?* LED in the *Control Indicators* section turns bright green. **Ensure the encoder cable does not interfere with the pendulum arm motion.**
8. Vary Offset and observe the *Arm Angle (deg)* response in the *Angle/Energy (deg/mJ)* scope. **Do not set the *Offset* too high or the encoder cable will interfere with the pendulum arm motion.**
9. As the pendulum is being balanced, describe the red *Arm Angle (deg)* and the blue *Pendulum Angle (deg)* responses in the *Angle/Energy (deg/mJ)* scope.
10. In the *Signal Generator* section set:
    - Amplitude = 45.0 deg
    - Frequency = 0.10 Hz
    - Offset = 0.0 deg
11. Observe the behaviour of the system when a square wave command is given to the arm angle. Why does the arm initially move in the wrong direction?
12. Click on the *Stop* button to stop running the VI.

4.4 Implement Designed Balance Control [20 min]

1. Go through Section 3.4 and design a balance control according to the given specifications.
   Remark: It is recommended to use the experimental determined pendulum moment of inertia that was found in Section 2.5.
2. Open the QNET_ROTPENT_Swing_Up_Control.vi and ensure it is configured as described in Section 6. **Make sure the correct *Device* is chosen.**
3. Run the QNET_ROTPENT_Swing_Up_Control.vi. The VI should appear similarity as shown in Figure 4.1.
4. In the *Signal Generator* section set:
    - Amplitude = 45.0 deg
    - Frequency = 0.20 Hz
    - Offset = 0.0 deg
5. To implement your balance controller, enter the control gain found in Section 3.4 in *kp_theta*, *kp_alpha*, *kd_theta*, and *kd_alpha* in the *Control Parameters* section.
6. Manually rotate the pendulum in the upright position until the *In Range?* LED in the *Control Indicators* section turns bright green. **Ensure the encoder cable does not interfere with the pendulum arm motion.**
7. Attach the response found *Angle/Energy (deg/mJ)* and the *Voltage (V)* scopes. Does your system meet the specifications given in Section 3.4?
8. Click on the *Stop* button to stop running the VI.

4.5 Balance Control with Friction Compensation [30 min]

1. Go through steps 1-7 in Section 4.3 to run the default balance control. The pendulum should be balancing.
2. In the *Signal Generator* section set:
    - Amplitude = 0.0 deg
    - Frequency = 0.10 Hz

- Offset = 0.0 deg

3. In the *Dither Signal* section set:
   - Amplitude = 0.00 V
   - Frequency = 2.50 Hz
   - Offset = 0.00 V
4. Observe the behaviour of *Arm Angle (deg)* in the *Angle/Energy (deg/mJ)* scope. Intuitively speaking, can you find some reasons why the arm is oscillating?
5. Increase the *Amplitude* in the *Dither Signal* section by steps of 0.1 V until you notice a change in the arm angle response.
6. From the *Voltage (V)* scope and the pendulum motion, what is the Dither signal doing? Compare the response of the arm with and without the Dither signal.
7. Increase the *Frequency* in the *Dither Signal* section starting from 1.00 to 10.0 Hz.
8. How does this effect the pendulum arm response?
9. Set the *Dither Signal* properties according to the friction measured in Section 2.4. How does this effect the pendulum arm response?
10. Click on the *Stop* button to stop running the VI.

5 SWING-UP CONTROL

5.1 Background

5.1.1 Energy Control

If the arm angle is kept constant and the pendulum is given an initial position it would swing with constant amplitude. Because of friction there will be damping in the oscillation. The purpose of energy control is to control the pendulum in such a way that the friction is constant. The potential energy of the pendulum is

$$E_p = M_p\, g\, l_p\, (1 - \cos\alpha) \tag{5.1}$$

and the kinetic energy is

$$E_k = \frac{1}{2} J_p \dot{\alpha}^2. \tag{5.2}$$

The potential energy is zero when the pendulum is at rest at $\alpha = 0$ in Figure 2.2, and equals $2M_p\, g\, l_p$ when the pendulum is upright at $\alpha = \pm\pi$. The sum of the potential and kinetic energy of the pendulum is

$$E = \frac{1}{2} J_p \dot{\alpha}^2 + M_p\, g\, l_p\, (1 - \cos\alpha)\,. \tag{5.3}$$

Differentiating 5.3 results in the differential equation

$$\dot{E} = \dot{\alpha}\left(J_p \ddot{\alpha}^2 + M_p\, g\, l_p \sin\alpha\right). \tag{5.4}$$

Substituting the pendulum equation of motion given in Equation 2.1 for pendulum acceleration into Equation 5.4 gives

$$\dot{E} = M_p\, u\, l_p\, \dot{\alpha} \cos\alpha.$$

Since the acceleration of the pivot is proportional to current driving the arm motor and thus also proportional to the drive voltage we find that it is easy to control the energy of the pendulum. The proportional control law

$$u = (E_r - E)\, \dot{\alpha} \cos\alpha \tag{5.5}$$

drives the energy towards the reference energy E_r. Notice that the control law is nonlinear because the proportional gain depends on the pendulum angle, α. Also, notice that the control changes sign when $\dot{\alpha}$ changes sign and when the angle is $\pm$ 90 deg.

However, for energy to change quickly the magnitude of the control signal must be large. As a result the following swing-up controller is implemented in the LabVIEW VI

$$u = \text{sat}_{u_{max}}\left(\mu(E_r - E)\text{sign}(\dot{\alpha}\cos\alpha)\right) \tag{5.6}$$

where μ is a tunable control gain and the $\text{sat}_{u_{max}}$ function saturates the control signal at the maximum acceleration of the pendulum pivot, u_{max}.

See Wikipedia for more information on potential energy, kinetic energy, control theory, and nonlinear control.

5.1.2 Hybrid Swing-Up Control

The energy swing-up control in 5.5 (or 5.6 can be combined with the balancing control law in 4.1 to obtain a control law which performs the dual tasks of swinging up the pendulum and balancing it. As illustrated in Figure 5.1, this can be accomplished by switching between the two control systems.

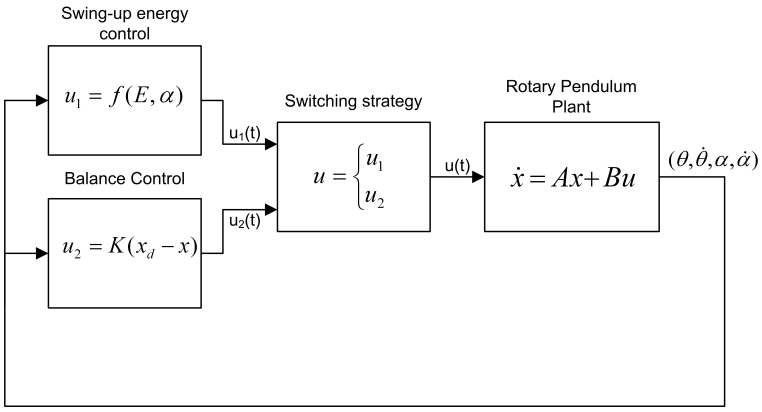

Figure 5.1: Swing-up hybrid control

This system can be modeled as a hybrid system. Hybrid systems are systems with both continuous and discrete parts. There are two continuous part: the closed-loop system using the swing-up energy controller and the closed-loop system using the PD balance controller. The switching strategy is the discrete element that chooses which controller, or system, to run. The switching logic can be obtained by determining a region in state space where the balancing works well. Balancing control is then used inside this region and energy control is used outside the region. Figure 5.2 is a called a hybrid automaton and, for this specific task, can be used to describe the system model and the switching logic.

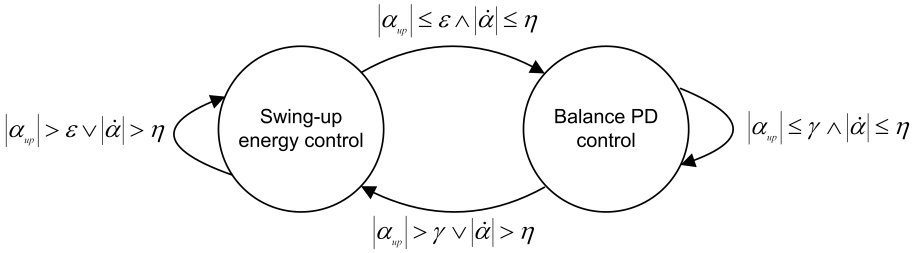
$|\alpha_{up}| \le \varepsilon \wedge |\dot{\alpha}| \le \eta$
$|\alpha_{up}| > \varepsilon \vee |\dot{\alpha}| > \eta$
Swing-up energy control
Balance PD control
$|\alpha_{up}| \le \gamma \wedge |\dot{\alpha}| \le \eta$
$|\alpha_{up}| > \gamma \vee |\dot{\alpha}| > \eta$

Figure 5.2: Hybrid swing-up controller automaton

The circles in Figure 5.2 are called *locations* and represent the two different continuous system. The arrows are called *edges* and represent the discrete jumps taken when certain condition are satisfied. The angle used in the switching logic in Figure 5.2 is called the upright angle. It is defined as zero when the pendulum is about its upright vertical position and expressed mathematically using

$$\alpha_{up} = \alpha \text{ mod } 2\pi - \pi.$$

The various switching parameters shown in Figure 5.2 can then be set as:

$$\begin{aligned} \epsilon &= 2 \text{ deg} \\ \eta &= 720 \text{ deg/s} \\ \gamma &= 30 \text{ deg} \end{aligned}$$

Given that the pendulum starts in the downward vertical position, it is in the swing-up location of the hybrid automaton. The swing-up controller pumps energy into the pendulum until it swings within $\pm$ 2 deg of its upright vertical position. Once the pendulum is within that that range and does not exceed 720 deg/s in either direction, the edge is taken to engage the balance controller. It remain in the *Balance PD control* location until the pendulum goes beyond the $\pm$ 30 deg position range or beyond $\pm$ 720 deg/s.

5.2 Swing-Up Control VI

The virtual instrument used to run the swing-up controller on the QNET rotary pendulum system is the same as the balance control given in Section 4.2 shown in Figure 4.1.

5.3 Energy Control [30 min]

1. Open the QNET_ROTPENT_Swing_Up_Control.vi and ensure it is configured as described in Section 6. **Make sure the correct *Device* is chosen.**
2. Run the QNET_ROTPENT_Swing_Up_Control.vi. The VI should appear similarity as shown in Figure 4.1.
3. In the *Balance Control Parameters* section ensure the following parameters are set:
   - kp_theta = -6.50 V/rad
   - kp_alpha = 80.0 V/rad
   - kd_theta = -2.75 V/(rad/s)
   - kd_alpha = 10.5 V/(rad/s)
4. In the *Swing-Up Control Parameters* section set:
   - mu = 55 $m/s^2/J$
   - Er = 20.0 mJ
   - max accel = 10 m/s^2
   - Activate Swing-Up = OFF (de-pressed)
5. Adjust the *Angle/Energy (deg/mJ)* scope scales to see between -250 and 250 (see the ROTPEN User Manual ([2]) for help).
6. Manually rotate the pendulum at different levels and examine the blue *Pendulum Angle (deg)* and the green *Pendulum Energy (mJ)* in the *Angle/Energy (deg/mJ)* scope. The pendulum energy is also displayed numerically in the *Control Indicators* section.
7. What do you notice about the energy when the pendulum is moved at different positions? Record the energy when the pendulum is being balanced (i.e., fully inverted in the upright vertical position).
8. Click on the *Stop* button to bring the pendulum down to the gantry position and re-start the VI.
9. In the *Swing-Up Control Parameters* section, turn ON the *Activate Swing-Up* switch (the pressed down position).
10. If the pendulum is not moving, click on the *Disturbance* button in the *Signal Generator* section to perturb the pendulum.
11. In *Swing-Up Control Parameters*, change the reference energy *Er* between 5.0 mJ and 50.0 mJ. As it is varied, examine the control signal in the *Voltage (V)* scope as well as the blue *Pendulum Angle (deg)* and the red *Pendulum Energy (mJ)* in the *Angle/Energy (deg/mJ)* scope. Attach the response of the *Angle/Energy (deg/mJ)* and *Voltage (V)* scopes.
12. In *Control Parameters* fix *Er* to 20.0 mJ and vary the swing-up control gain *mu* between 10 and 100 $m/s^2/J$. Describe how this changes the performance of the energy control.
13. Click on the *Stop* button to stop running the VI.

5.4 Hybrid Swing-Up Control [20 min]

1. Open the QNET_ROTPENT_Swing_Up_Control.vi and ensure it is configured as described in Section 6. **Make sure the correct *Device* is chosen.**
2. Run the QNET_ROTPENT_Swing_Up_Control.vi. The VI should appear similarity as shown in Figure 4.1.
3. In the *Balance Control Parameters* section ensure the following parameters are set:
   - kp_theta = -6.50 V/rad
   - kp_alpha = 80.0 V/rad
   - kd_theta = -2.75 V/(rad/s)
   - kd_alpha = 10.5 V/(rad/s)
4. In the *Swing-Up Control Parameters* section set:
   - mu = 55 $m/s^2/J$
   - Er = 20.0 mJ
   - max accel = 10 m/s^2
   - Activate Swing-Up = OFF (de-pressed)
5. Adjust the *Angle/Energy (deg/mJ)* scope scales to see between -250 and 250 (see the ROTPEN User Manual ([2] for help).
6. Make sure the pendulum is hanging down motionless and the encoder cable is not interfering with the pendulum.
7. In the *Swing-Up Control Parameters*, set the *Activate Swing-Up* switch to ON (pressed down position).
8. The pendulum should begin going back and forth. If not, click on the *Disturbance* button in the *Signal Generator* section to perturb the pendulum. **Turn off the *Active Swing-Up* switch if the pendulum goes unstable or if the encoder cable interferes with the pendulum arm motion.**
9. Gradually increase the reference energy *Er* in the *Control Parameters* section until the pendulum swings up to the vertical position.
10. What reference energy was required to swing-up the pendulum? Was this value expected?
11. Click on the *Stop* button to stop running the VI.

6 SYSTEM REQUIREMENTS

Required Hardware

- NI ELVIS II (or NI ELVIS I)
- Quanser QNET Rotary Inverted Pendulum Trainer (ROTPENT). See QNET ROTPENT User Manual ([2]).

Required Software

- NI LabVIEW®2010 or later
- NI LabVIEW®Control Design and Simulation Module
- *ELVIS II Users*: NI ELVISmx (installs required NI DAQmx drivers)
- *ELVIS I Users*:
  - NI DAQmx
  - ELVIS CD 3.0.1 or later installed

■ **Caution: If these are not all installed then the VI will not be able to run!** Please make sure all the software and hardware components are installed. If an issue arises, then see the troubleshooting section in the QNET ROTPENT User Manual ([2]).

6.1 Overview of Files

File Name	Description
QNET ROTPENT User Manual.pdf	This manual describes the hardware of the QNET Rotary Pendulum Trainer system and how to setup the system on the ELVIS.
QNET ROTPENT Workbook (Student).pdf	This laboratory guide contains pre-lab questions and lab experiments demonstrating how to design and implement controllers on the QNET DCMCT system LabVIEW®.
QNET_ROTPENT_Simple_Modeling.vi	Apply voltage to DC motor and examine the arm and pendulum responses.
QNET_ROTPENT_Control_Design.vi	Design and simulate LQR-based balance controller.
QNET_ROTPENT_Swing_Up_Control.vi	Swing-up and balance pendulum.

Table 2: Files supplied with the QNET ROTPENT Laboratory.

6.2 Simple Modeling Laboratory VI

The QNET-ROTPENT Simple Modeling VI is shown in Figure 6.1. It runs the DC motor connected to the pendulum arm in open-loop and plots the corresponding pendulum arm and link angles as well as the applied input motor voltage. Table 3 lists and describes the main elements of the ROTPENT Simple Modeling virtual instrument front panel. Every element is uniquely identified through an ID number and located in Figure 6.1.

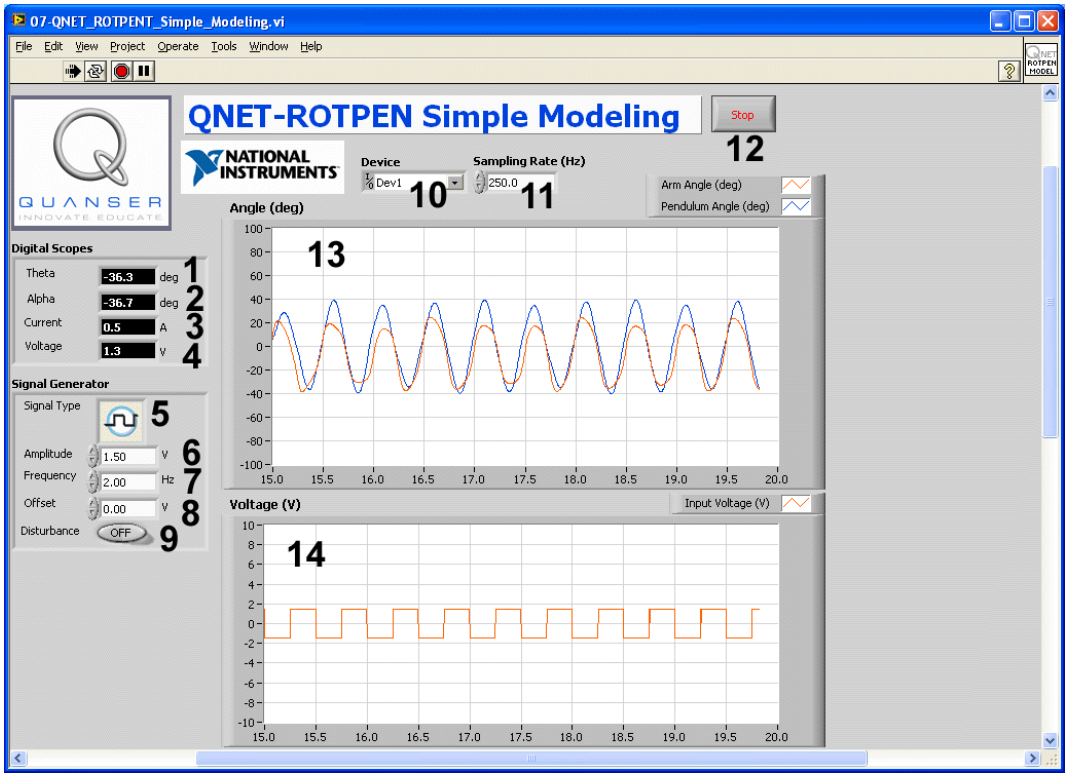

Figure 6.1: QNET-ROTPENT Simple Modeling virtual instrument.

6.3 Control Design VI

The QNET ROTPENT Control Design VI enables users to design a balance controller and simulate its response. The matrices for the state-space model of the rotary inverted pendulum system is shown in the *Symbolic Model* tab and illustrated in Figure 6.2. The values of the variables used in the state-space model can be changed. In the *Open Loop Analysis* tab, shown in Figure 6.3, the numerical state-space model is displayed and the resulting open-loop poles are plotted on a phase plane. Based on this model, a controller to balance the rotary inverted pendulum system can be designed using the Linear-Quadratic Regulator (LQR) optimization technique, as shown in the *Simulation* tab in Figure 6.4. The resulting closed-loop inverted pendulum system can be simulated. Table 4 lists and describes the main elements of the ROTPENT Control Design virtual instrument user interface. Every element is uniquely identified through an ID number and located in figures 6.2, 6.3, and 6.4.

6.4 Swing-Up Control VI

The QNET Rotary Pendulum Trainer Swing-Up Control VI implements an energy-based control that swings up the pendulum to its upright vertical position and a state-feedback controller to balance the pendulum when in its upright position. The main elements of the VI front panel are summarized in Table 5 and identified in Figure 6.5 through the corresponding ID number.

ID #	Label	Symbol	Description	Unit
1	Theta	θ	Arm angle numeric display measured by encoder on motor.	deg
2	Alpha	α	Pendulum angle numeric display measured by encoder on pendulum pivot.	deg
3	Current	I_m	Motor armature current numeric display.	A
4	Voltage	V_m	Motor input voltage numeric display.	V
5	Signal Type		Type of signal generated for the input voltage signal.	
6	Amplitude		Generated signal amplitude input box.	V
7	Frequency		Generated signal frequency input box.	Hz
8	Offset		Generated signal offset input box.	V
9	Disturbance	V_{sd}	Apply simulated disturbance voltage.	V
10	Device		Selects the NI DAQ device.	
11	Sampling Rate		Sets the sampling rate of the VI.	Hz
12	Stop		Stops the LabVIEW VI from running.	
13	Scopes: Angle	θ, α	Scope with measured arm angle (in red) and pendulum angle (in blue).	deg
14	Scopes: Voltage	V_m	Scope with applied motor voltage (in red).	V

Table 3: QNET ROTPENT Simple Modeling VI Components

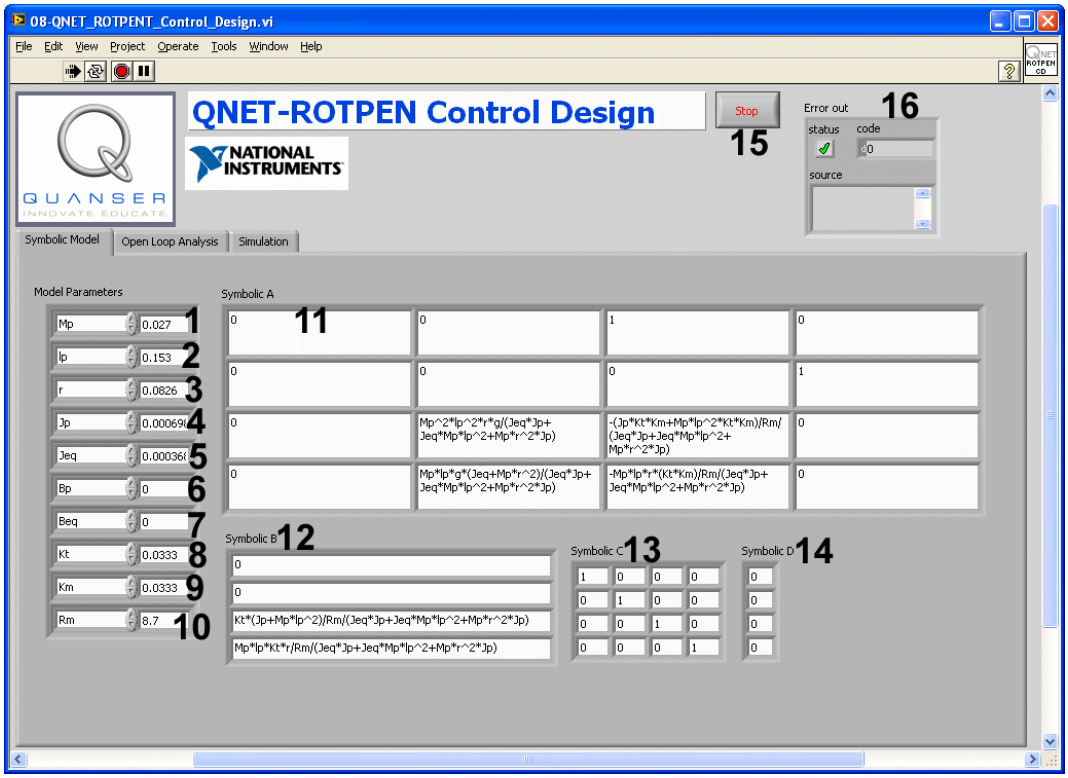

Figure 6.2: QNET ROTPENT Control Design VI: *Symbolic Model* tab.

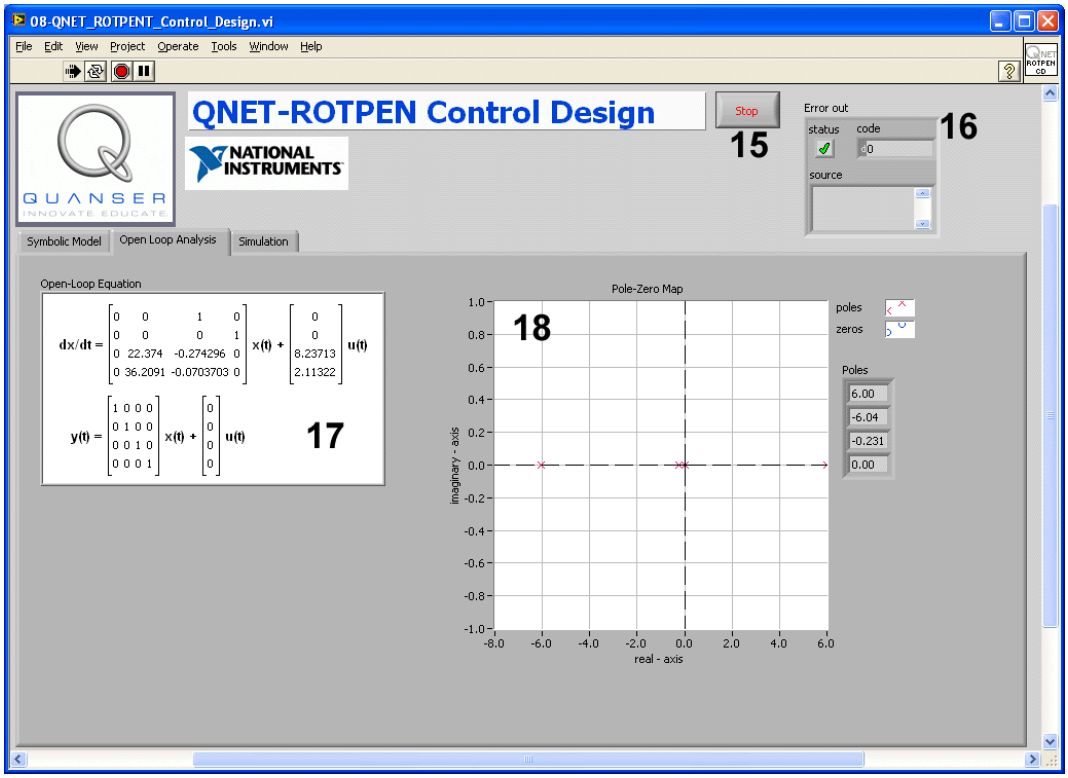

Figure 6.3: QNET ROTPENT Control Design VI: *Open Loop Analysis* tab.

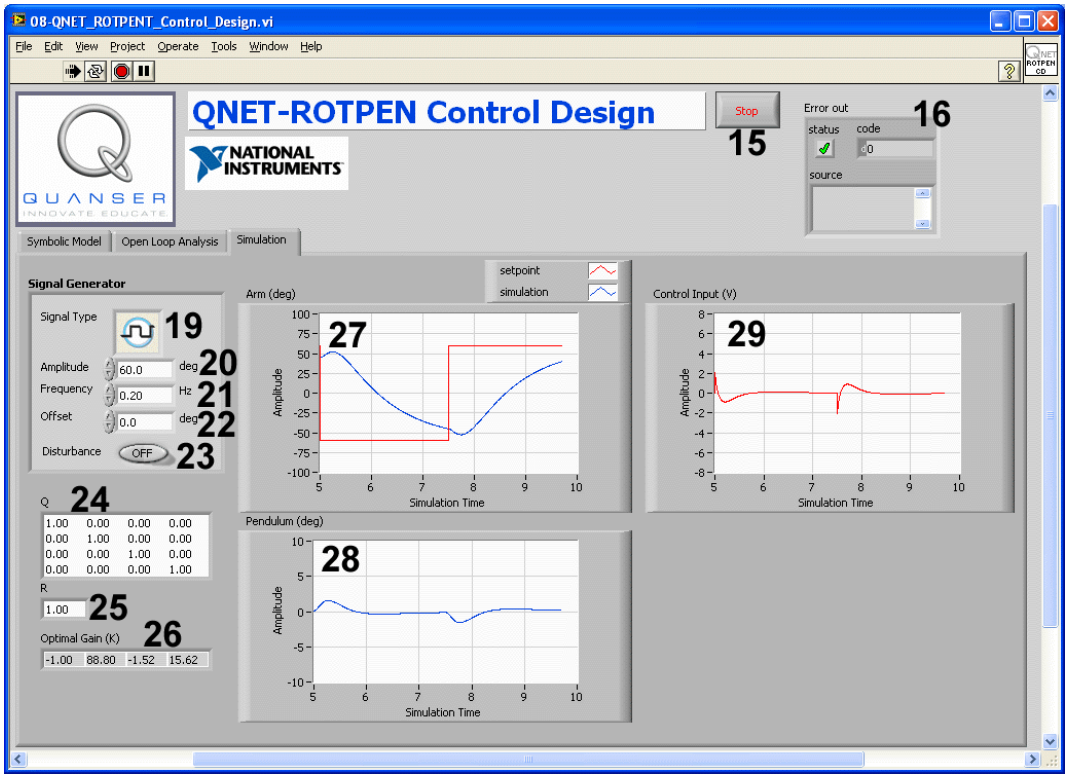

Figure 6.4: QNET ROTPENT Control Design VI: *Simulation*tab.

ID #	Label	Symbol	Description	Unit
1	Mp	M_p	Mass of pendulum assembly (link + weight).	kg
2	lp	l_p	Center of mass of pendulum assembly (link+weight) input box.	m
3	r	r	Length from motor shaft to pendulum pivot.	m
4	Jp	J_p	Pendulum moment of inertia relative to pivot.	kg.m2
5	Jeq	J_{eq}	Equivalent moment of inertia acting on the DC motor shaft.	kg.m2
6	Bp	B_p	Viscous damping about the pendulum pivot.	N.m.s/rad
7	Beq	B_{eq}	Equivalent viscous damping acting on the DC motor shaft.	N.m.s/rad
8	Kt	K_t	DC motor current-torque constant.	N.m/A
9	Km	K_m	DC motor back-emf constant.	V.s/rad
10	Rm	R_m	Electrical resistance of the DC motor armature.	Ω
11	Symbolic A	A	Rotary pendulum linear state-space matrix A.	
12	Symbolic B	B	Rotary pendulum linear state-space matrix B.	
13	Symbolic C	C	Rotary pendulum linear state-space matrix C.	
14	Symbolic D	D	Rotary pendulum linear state-space matrix D.	
15	Stop		Stops the LabVIEW VI from running.	
16	Error Out		Displays any error encountered in the VI.	
17	Open-Loop Equation		Numeric linear state-space model of rotary pendulum.	
18	Pole-Zero Map		Maps pole and zeros of open-loop rotary pendulum system.	
19	Signal Type		Type of signal generated for the arm position reference.	
20	Amplitude		Generated signal amplitude input box.	V
21	Frequency		Generated signal frequency input box.	Hz
22	Offset		Generated signal offset input box.	V
23	Disturbance	V_{sd}	Apply simulated disturbance voltage.	V
24	Q	Q	Linear-quadratic weighting matrix that defines a penalty on the state.	
25	R	R	Linear-quadratic weighting matrix that defines a penalty on the control action.	
26	Optimal Gain (K)	K	State-feedback control gain calculated using LQR.	
27	Arm	θ	Scope with reference (in blue) and measured (in red) arm angles.	deg
28	Pendulum	α	Scope with inverted pendulum angle (in blue).	deg
29	Control Input	V_m	Scope with applied motor voltage (in red).	V

Table 4: QNET ROTPENT Control Design VI Components

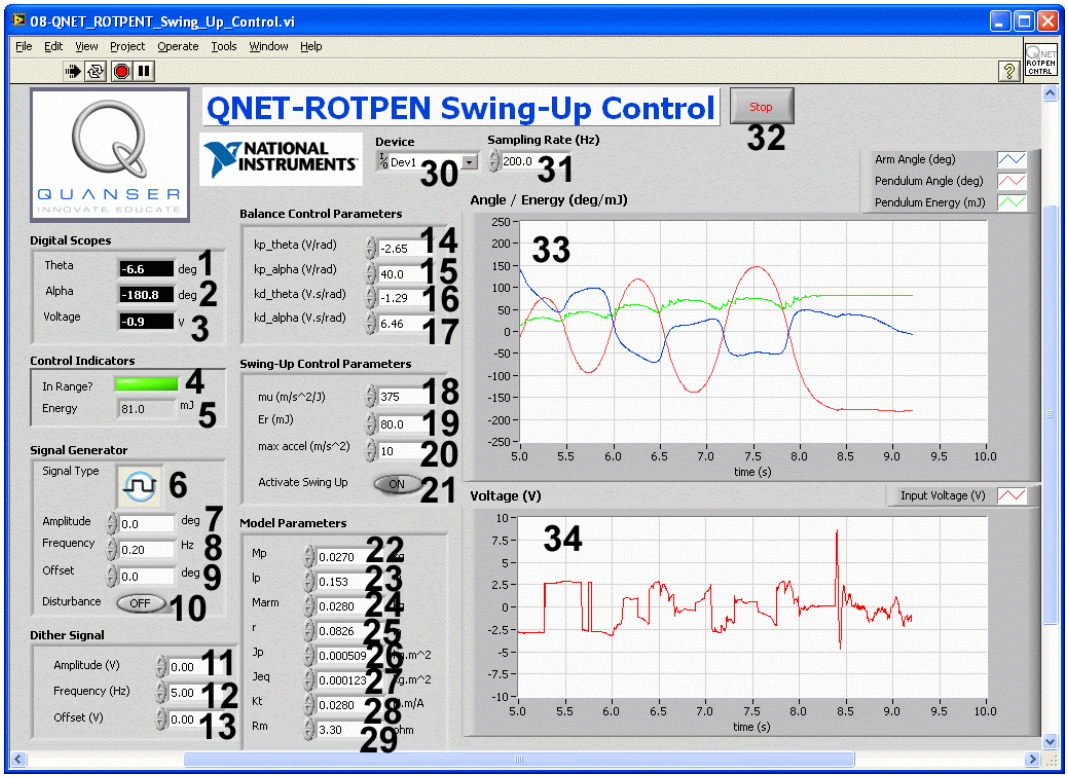

Figure 6.5: QNET ROTPENT Swing-Up Control VI.

ID #	Label	Symbol	Description	Unit
1	Theta	θ	Arm angle numeric display measured by encoder on motor.	deg
2	Alpha	α	Pendulum angle numeric display measured by encoder on pendulum pivot.	deg
3	Current	I_m	Motor armature current numeric display.	A
4	In Range?		Balance controller is engaged when this LED is turns bright green.	
5	Energy		Numeric display of the pendulum energy.	mJ
6	Signal Type		Type of signal generated for the arm reference signal (i.e., desired angle of arm).	
7	Amplitude		Reference position amplitude input box.	deg
8	Frequency		Reference position frequency input box.	Hz
9	Offset		Reference position offset input box.	deg
10	Disturbance	V_{sd}	Apply simulated disturbance voltage.	V
11	Amplitude	A_d	Dither signal amplitude input box.	V
12	Frequency	f_d	Dither signal frequency input box.	Hz
13	Offset	V_{d0}	Dither signal offset input box.	V
14	kp_theta	$k_{p,\theta}$	Arm angle proportional gain input box.	V/rad
15	kp_alpha	$k_{p,\alpha}$	Pendulum angle proportional gain input box.	V/rad
16	kd_theta	$k_{d,\theta}$	Arm angle derivative gain input box.	V.s/rad
17	kd_alpha	$k_{d,\alpha}$	Pendulum angle derivative gain input box.	V.s/rad
18	mu	μ	Proportional gain for energy controller.	m/(s2.J)
19	Er	E_r	Reference energy for energy controller.	mJ
20	Max accel	u_{max}	Maximum acceleration	m/s^2
21	Activate Swing Up		When pressed down the energy controller that swings-up the pendulum is engaged.	
22	Mp	M_p	Mass of pendulum assembly (link + weight).	kg
23	lp	l_p	Center of mass of pendulum assembly (link+weight) input box.	m
24	Marm	M_{arm}	Mass of rotary arm.	kg
25	r	r	Length from motor shaft to pendulum pivot.	m
26	Jp	J_p	Pendulum moment of inertia relative to pivot.	kg.m2
27	Jeq	J_{eq}	Equivalent moment of inertia acting on the DC motor shaft.	kg.m2
28	Kt	K_t	DC motor current-torque constant.	N.m/A
29	Rm	R_m	Electrical resistance of the DC motor armature.	Ω
30	Device		Selects the NI DAQ device.	
31	Sampling Rate		Sets the sampling rate of the VI.	Hz
32	Stop		Stops the LabVIEW VI from running.	
33	Scopes: Angle	θ, α	Scope with measured arm angle (in red) and pendulum angle (in blue).	deg
34	Scopes: Voltage	V_m	Scope with applied motor voltage (in red).	V

Table 5: QNET ROTPENT Swing-Up Control VI Components

7 LAB REPORT

This laboratory contains three groups of experiments, namely,

1. Modeling,
2. Balance control design
3. Balance control implementation
4. Swing-up control

For each experiment, follow the outline corresponding to that experiment to build the *content* of your report. Also, in Section 7.5 you can find some basic tips for the *format* of your report.

7.1 Template for Content (Simple Modeling)

I. PROCEDURE

1. *Damping*
   - Briefly describe the main goal of the experiment.
   - Briefly describe the experiment procedure in Step 6 in Section 2.3.
2. *Friction*
   - Briefly describe the main goal of the experiment.
   - Briefly describe the experiment procedure in Step 5 in Section 2.4.
3. *Moment of Inertia*
   - Briefly describe the main goal of the experiment.
   - Briefly describe the experiment procedure in Step 4 in Section 2.5.

II. RESULTS
Do not interpret or analyze the data in this section. Just provide the results.

1. Provide applicable data collected in this laboratory from Table 1.

III. ANALYSIS
Provide details of your calculations (methods used) for analysis for each of the following:

1. Damping analysis in step 6 in Section 2.3.
2. Finding friction in step 5 in Section 2.4.
3. Calculating moment of inertia of pendulum in step 4 in Section 2.5.

IV. CONCLUSIONS
Interpret your results to arrive at logical conclusions for the following:

1. How well does the experimentally derived moment of inertia compare with analytically derived value in step 5 of Section 2.5.

7.2 Template for Content (Balance Control Design)

I. PROCEDURE

1. *Model Analysis*
    - Briefly describe the main goal of the experiment.
    - Briefly describe the experimental procedure in Step 6 in Section 3.3.
2. *Control Design and Simulation*
    - Briefly describe the main goal of the experiment.
    - Briefly describe the experimental procedure in Step 6 in Section 3.4.

II. RESULTS
Do not interpret or analyze the data in this section. Just provide the results.

1. LQR matrices and control gain found in Step 9 in Section 3.4.
2. Simulated closed-loop response plot from Step 10 in Section 3.4.

III. ANALYSIS
Provide details of your calculations (methods used) for analysis for each of the following:

1. Open-loop poles in Step 6 in Section 3.3.
2. Effect of changing moment of inertia on open-loop poles in Step 8 in Section 3.3.
3. Effect of changing LQR elements on response in Step 6 in Section 3.4.
4. Effect of changing different LQR element on the response in Step 8 in Section 3.4.

IV. CONCLUSIONS
Interpret your results to arrive at logical conclusions for the following:

1. Does lowering the moment of inertia of the pendulum have the expected result Step 8 in Section 3.3.
2. Does the simulation match the specifications in Step 10 in Section 3.4.

7.3 Template for Content (Balance Control Implementation)

I. PROCEDURE

1. *Default Balance Control*
   - Briefly describe the main goal of the experiment.
   - Briefly describe the experimental procedure in Step 8 in Section 4.3.
2. *Implement Designed Balance Control*
   - Briefly describe the main goal of the experiment.
   - Briefly describe the experimental procedure in Step 7 in Section 4.4.
3. *Balance Control with Friction Compensation*
   - Briefly describe the main goal of this experiment.
   - Briefly describe the experimental procedure in Step 4 in Section 4.5.

II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

1. Balance control response plot from step 7 in Section 4.4.

III. ANALYSIS

Provide details of your calculations (methods used) for analysis for each of the following:

1. Effect of changing offset in Step 8 in Section 4.3.
2. Balance control analysis in Step 9 in Section 4.3.
3. Balance control analysis when tracking step reference in 11 in Section 4.3.
4. Examining the arm oscillation in Step 4 in Section 4.5.
5. Explain what the Dither signal is doing in Step 6 in Section 4.5.
6. Effect of increasing Dither signal frequency in Step 8 in Section 4.5.

IV. CONCLUSIONS

Interpret your results to arrive at logical conclusions for the following:

1. Whether the balance controller meets the specifications in Step 7 in Section 4.4.
2. Effect of setting the Dither signal to the identified friction parameters in Step 9 of Section 4.5.

7.4 Template for Content (Swing-Up Control)

I. PROCEDURE

1. *Energy Control*
    - Briefly describe the main goal of the experiment.
    - Briefly describe the experimental procedure in Step 7 in Section 5.3.
2. *Hybrid Swing-Up Control*
    - Briefly describe the main goal of the experiment.
    - Briefly describe the experimental procedure in Step 9 in Section 5.4.

II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

1. Pendulum response from Step 11 in Section 5.3.

III. ANALYSIS

Provide details of your calculations (methods used) for analysis for each of the following:

1. Energy at different pendulum position in Step 7 in Section 5.3.
2. Effect of changing reference energy in Step 11 in Section 5.3.
3. Effect of changing proportional gain in Step 11 in Section 5.3.

IV. CONCLUSIONS

Interpret your results to arrive at logical conclusions for the following:

1. Reference energy required to swing-up pendulum in Step 10 of Section 5.4.

7.5 Tips for Report Format

PROFESSIONAL APPEARANCE

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- Typed.
- All grammar/spelling correct.
- Report layout is neat.
- Does not exceed specified maximum page limit, if any.
- Pages are numbered.
- Equations are consecutively numbered.
- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- No hand drawn sketches/diagrams.
- References are cited using correct format.

REFERENCES

[1] Quanser Inc. *QNET Rotary Pendulum Control Trainer User Manual*, 2011.

Six QNET Trainers to teach introductory controls using NI ELVIS

QNET DC Motor Control Trainer
teaches fundamentals of DC motor control

QNET HVAC Trainer
teaches temperature (process) control

QNET Mechatronic Sensors Trainer
teaches functions of 10 different sensors

QNET Rotary Inverted Pendulum Trainer
teaches classic pendulum control experiment

QNET Myoelectric Trainer
teaches control using principles of electromyography (EMG)

QNET VTOL Trainer
teaches basic flight dynamics and control

NI ELVIS I

NI ELVIS II

Quanser QNET Trainers are plug-in boards for NI ELVIS to teach introductory controls in undergraduate labs. Together they deliver added choice and cost-effective teaching solutions to engineering educators. All six QNET Trainers are offered with comprehensive, ABET-aligned course materials that have been developed to enhance the student learning experience.

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